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Efimov Physics in small bosonic clusters

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Abstract We study small clusters of bosons, $A = 2, 3, 4, 5, 6$, characterized by a resonant interaction. Firstly, we use a soft-gaussian interaction that reproduces the values of the dimer binding energy and the atom-atom scattering length obtained with LM2M2 potential, a widely used ^{4}He-^{4}He interaction. We change the intensity of the potential to explore the clusters' spectra in different regions with large positive and large negative values of the two-body scattering length and we report the clusters' energies on Efimov plot, which makes the scale invariance explicit. Secondly, we repeat our calculation adding a repulsive three-body force to reproduce the trimer binding energy. In all the region explored, we have found that these systems present two states, one deep and one shallow close to the $A-1$ threshold, and scale invariance has been investigated for these states. The calculations are performed by means of Hyperspherical Harmonics basis set.

Keywords Efimov Physics · Bosonic Clusters · Hyperspherical Harmonics

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1 Introduction

Systems composed by few particles having large value of the two-body scattering length, a, with respect to the natural length, ℓ, fixed by the inter-particle potential, have been the object of an intense investigation both from a theoretical and experimental point of view (for recent reviews see Refs. [1, 2, 3]). The interest is driven by their

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universal properties; the behavior of observables do not depend on the microscopical characteristics, namely the inter-particle potential, but only on symmetries.

In the limit $a/\ell \to \infty$, known either as resonant $a \to \infty$ or as scaling $\ell \to 0$ limit, the few-particle systems display discrete-scale symmetry as brought out by Efimov in his works in the early 70's [4,5]. The scaling factor is usually written as $\lambda = \mathrm{e}^{\pi/s_0}$, and for three-identical bosons $s_0 = 1.00624$ and $\lambda \approx 22.69$. This symmetry implies that all observables can be written as an universal log-period function of the dimensionless variable $a\kappa_*$, where κ_* is a three-body parameter encoding the high-energy (short distance) physics, which enters, at leading order, only through this parameter. The fact that the observables' behavior is governed by discrete-scale symmetry is know as Efimov physics. For $A = 3$, these properties have been studied for large positive and large negative values of the scattering length in the (a^{-1}, k) plane, with $k = \mathrm{sign}(E)[|E|/(\hbar^2/m)]^{1/2}$, constructing the Efimov plot [5]. This plot is useful in the identification of discrete-scale symmetry; in fact, if one introduce the radial, H, and angular, ξ Efimov variables by $k = H\sin\xi$ and $a^{-1} = H\cos\xi$, the scale invariance reads $H \to \lambda^{-1}H$, and $\xi \to \xi$.

In the present work we extend our previous analysis of the $A = 4-6$ bosonic spectrum [6] to the (a^{-1}, k) plane. We have modified the strength of the LM2M2 [7] potential in order to cover the region of negative values of a up to a_-^0, with this value indicating the threshold of having a three-body system bound. We have also increased the intensity of the interaction in order to extend the analysis to positive values of a in which the universal character of the system starts to be questionable, i.e, when the ground-state E_3^0 approaches the natural energy $E_\ell = -\hbar^2/m\ell^2$, which delimits the Efimov window.

We used the LM2M2 potential to fix the two-body soft-core potential as in discussed in Refs. [8,6]; this has been possible because of the scale separation between the ^{4}He-^{4}He scattering length, $a = 189.41$ a.u., and the natural length $\ell = 10.2$ a.u., which is the van der Waals length calculated for the LM2M2 potential [1]. In the three-body sector, a three-body soft-core potential is required to reproduce the ground-state-binding energy of the helium trimer given by the LM2M2 potential.

We have performed our numerical calculations in systems with $A \geq 3$, by means of the non-symmetrized hyperspherical harmonic (NSHH) expansion method with the technique recently developed by the authors in Refs. [9,10,11,12]. In this approach, the authors have used the Hyperspherical Harmonic (HH) basis, without a previous symmetrization procedure, and on the representation of the Hamiltonian matrix, as a sum of products of sparse matrices, well suited for a numerical implementation.

As a result, we have observed that in all the region explored the $A = 4, 5, 6$ systems present two states, one deep and one shallow close to the E_{A-1}^0 threshold. This analysis confirms, at least in one zone of the Efimov plot, previous observations that each Efimov state in the $A = 3$ system produces two bound states in the $A = 4$ system, and extends this observation to the $A = 5, 6$ systems.

2 Potentials

In our calculation we used $\hbar^2/m = 43.281307$ (a.u.)^2 K as mass parameter. The LM2M2 interaction has been modified in the following way

$$V_\lambda(r) = \lambda \cdot V_{\mathrm{LM2M2}}(r)\,, \tag{1}$$

and we have varied λ from $\lambda = 0.883$, where $a = a_-^0 = -43.84$ a.u., up to $\lambda = 1.1$ corresponding to $a = 44.79$ a.u. The unitary limit is produced for $\lambda \approx 0.9743$.

The two-body gaussian (TBG) potential is

$$V(r) = V_0 \, \mathrm{e}^{-r^2/R_0^2} \,, \tag{2}$$

with range $R_0 = 10$ a.u., and we have varied the strength V_0 in order to reproduce the values of a given by $V_\lambda(r)$. In the three-body sector we need an hypercentral-three-body (H3B) interaction to better describe the $A = 3$ system obtained with the modified LM2M2 potential

$$W(\rho_{123}) = W_0 \, \mathrm{e}^{-\rho_{123}^2/\rho_0^2} \,, \tag{3}$$

with the strength W_0 tuned to reproduce the trimer energy E_3^0 obtained using $V_\lambda(r)$. Here $\rho_{123}^2 = \frac{2}{3}(r_{12}^2 + r_{23}^2 + r_{31}^2)$ is the hyperradius of three particles and the range of the three-body force $\rho_0 = R_0$

3 Results

We have solved the $A = 3$ problem for bound states using the modified LM2M2 potential, and then we used the resulting energies to fix the strength of the H3B force. Than we have diagonalized the Hamiltonian for $A = 3, 4, 5, 6$ bodies using the TBG and TBG+H3B potentials. The results are given in Fig. 1 in two (a^{-1}, k) plots, which have been scaled to shrink the scale factor to $\sqrt{\lambda} \approx 4.8$.

In the case with only the TBG potential, upper panel of Fig. 1, we observe that the spectrum of the systems $A = 4, 5, 6$ presents two bound states, one deep and one shallow, for all values of a studied. These calculations confirms the prediction for $A = 4$ of a pair of tetramers attached to a trimer [13,14,15], and extends the observation to $A = 5$ and $A = 6$ systems.

When the repulsive three-body force is included, lower panel of Fig. 1, the spectrum moves up and we can observe that the excited state E_A^1 disappears for $A = 5, 6$ for negative values of the scattering length as a approaches a_-^1. This behaviour is sensitive to the range of the three-body force ρ_0, and it has been deeply investigated in Ref. [16].

In Fig. 2 we investigate the scale invariance using Efimov-polar coordinates: for a fixed value of ξ we report the ratio $(E_A^1/E_A^0)^{1/2}$ calculated with TBG potential. For negative values of a, the ratios tend to be constants in agreement with discrete-scale invariance; we note that, even if the ratio for $A = 3$ tends to be constant, the value is lower than the expected λ. For $a > 0$, which corresponds to the shaded zone in Fig. 2, the non-universal behaviour becomes stronger, and the ratios are no more constants.

To sum up, we have shown that Efimov physics for $A > 3$ manifests with the existence of two states for $A = 4, 5, 6$, and that the ratio between each pair tends to be constant as a function of the Efimov angle ξ.

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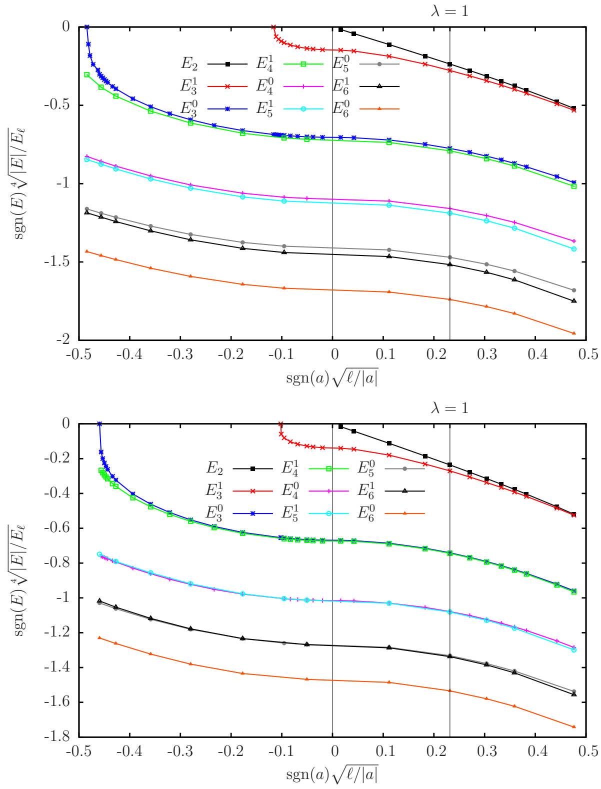

Fig. 1 (color online). Energies of the $A = 3 - 6$ ground and excited states, E_A^0, E_A^1, as a function of a^{-1}, using the two-body gaussian potential (upper panel), and using the two-body plus the hypercentral three-body force (lower panel). In both panels we also give the two-body ground-state energy E_2 calculated with the LM2M2 potential.

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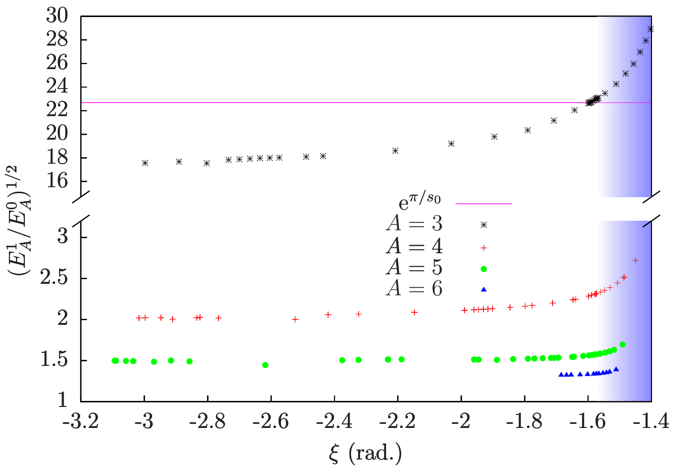

Fig. 2 (color online). Square root of the ratios between energies of A-particle systems as a function of Efimov angle ξ. The shaded region corresponds to positive scattering length.

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